Reg.No. ____________

Karunya UNIVERSITY

(Karunya Institute of Technology & Sciences)
(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

End Semester Examination – Nov/Dec – 2016

			Semester	**:**	**2016-17 ODD**
Code	**:**	**15MA3009**	**Duration**	**:**	**3hrs**
Sub. Name :		**Field Thoery**	**Max. marks :**		**100**

ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)

Q. No.	Sub Div.	Questions	Course Outcome	Marks
1.	a.	Prove that *every finite division ring is a field.*	COC1	**20**
		(OR)		
2.	a.	If F is a finite field and $\alpha \neq 0$ and $\beta \neq 0$ are two elements of F, then prove that there exist a and b in F such that $1+\alpha a^2 + \beta b^2 = 0$.	COC1	**14**
	b.	If the finite field F has p^m elements, the prove that every element a in F satisfies $a^{p^m} = a$.	COC1	**6**
3.	a.	If $f(x)$ and $g(x)$ are two non-zero elements of $F[x]$, then prove that $deg(f(x)g(x)) = deg\, f(x) + deg\, g(x)$.	COC2	**10**
	b.	Prove that any polynomial in $F[x]$ can be written in the unique manner as a product of irreducible polynomials in $F[x]$	COC2	**10**
		(OR)		
4.	a.	State and prove *Eisenstein criteria for polynomial* .	COC2	**10**
	b.	Prove that the product of two primitive polynomials is again primitive.	COC2	**10**
5.	a.	If K is a finite extension field of F and L is a finite extension of K, then prove that L is a finite extension of F and $[L:F] = [L:K][K:F]$	COC2	**20**
		(OR)		
6.	a.	If an element a in K is algebraic over F, then prove that there exists a unique monic polynomial $P(x)$ of positive degree over F such that i. $P(a) = 0$ ii. If for any polynomial $f(x)$ in $F[x]$ with $f(a) = 0$, then $P(x)$ divides $f(x)$.	COC2	**20**
7.	a.	Find the deg of the extension of the splitting field of $x^3 - 2 \in Q[x]$	COC2	**10**
	b.	State and prove the exsistence of splitting field for any non-constantpolynomial in $F[x]$, where F is a field.	COC2	**10**
		(OR)		
8.	a.	Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.	COC3	**20**
		Compulsory:		
9.	a.	State and prove the *fundamental theory of Galois group*	COC3	**20**

Course Outcomes: Students will be able to understand the proof techniques in

COC 1: Wedderburn Theorem on Finite Division Ring,

COC 2 : Eisenstein Irreducible Criterian,

COC 3: Solvability by radicals.

ALL THE BEST